
# ECS455: Chapter 5

## OFDM


Dr.Prapun Suksompong

www.prapun.com


**Office Hours:**

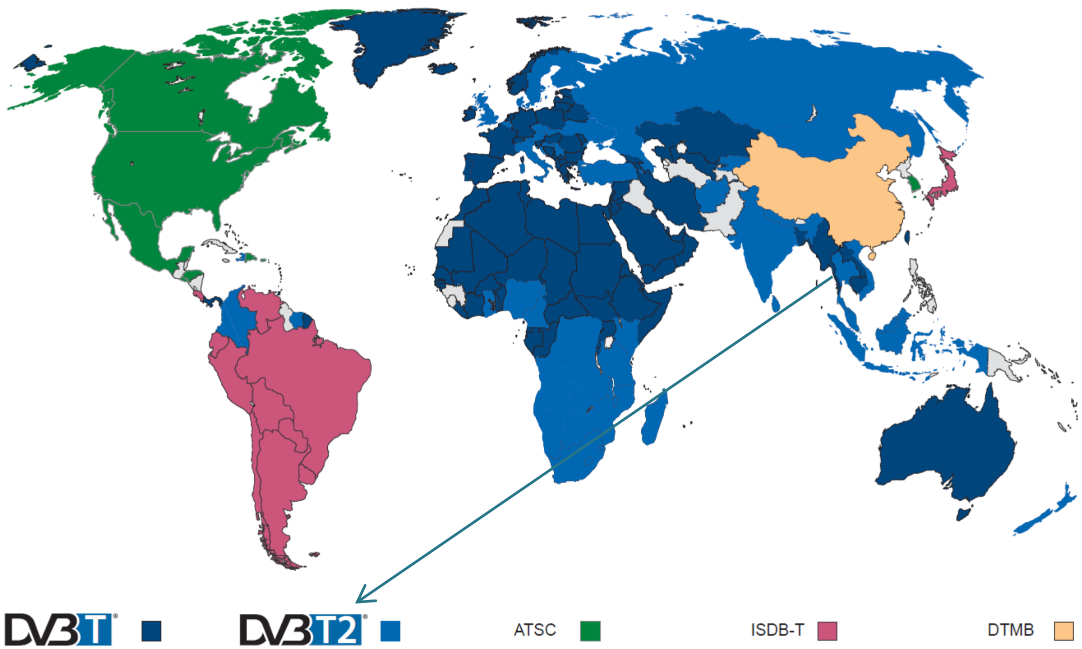
| | | |
|---|---|---|
| Library (Rangsit) | Mon | 16:20-16:50 |
| BKD 3601-7 | Wed | 9:20-11:20 |

# OFDM Applications

- 802.11 **Wi-Fi**: a/g/n/ac versions
- **DVB-T** (Digital Video Broadcasting — Terrestrial)
  - terrestrial digital TV broadcast system used in most of the world outside North America
- DMT (the standard form of **ADSL** - Asymmetric Digital Subscriber Line)
- **WiMAX, LTE (OFDMA)**

| Wireless | Wireline |
|---|---|
| IEEE 802.11a, g, n (WiFi) Wireless LANs | ADSL and VDSL broadband access via POTS copper wiring |
| IEEE 802.15.3a Ultra Wideband (UWB) Wireless PAN | MoCA (Multi-media over Coax Alliance) home networking |
| IEEE 802.16d, e (WiMAX), WiBro, and HiperMAN Wireless MANs | PLC (Power Line Communication) |
| IEEE 802.20 Mobile Broadband Wireless Access (MBWA) | |
| DVB (Digital Video Broadcast) terrestrial TV systems: DVB-T, DVB-H, T-DMB, and ISDB-T | |
| DAB (Digital Audio Broadcast) systems: EUREKA 147, Digital Radio Mondiale, HD Radio, T-DMB, and ISDB-TSB | |
| Flash-OFDM cellular systems | |
| 3GPP UMTS & 3GPP@ LTE (Long-Term Evolution) and 4G | |

# Side Note: Digital TV




Digital Video Broadcasting
– Second Generation Terrestrial

**Japan**: Starting July 24, 2011, the analog broadcast has ceased and only digital broadcast is available.

**US**: Since June 12, 2009, full-power television stations nationwide have been broadcasting exclusively in a digital format.

## Thailand's Roadmap:

# OFDM: Overview (1)

- Let $\underline{S} = (S_1, S_2, \ldots, S_N)$ contains the information symbols.

# OFDM: Overview (2)

- Let $\underline{S} = (S_1, S_2, \ldots, S_N)$ be the information symbol.
- The discrete baseband OFDM modulated symbol can be expressed as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left( j \frac{2\pi k t}{T_s} \right), \quad 0 \le t \le T_s$$

$$= \sum_{k=0}^{N-1} S_k \underbrace{\frac{1}{\sqrt{N}} 1_{[0,T_s]}(t) \exp\left( j \frac{2\pi k t}{T_s} \right)}_{c_k(t)}$$

Some references may use different constant in the front

Some references may start with different time interval, e.g. [-$T_s$/2, +$T_s$/2]

Note that:

$$\mathrm{Re}\{s(t)\} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left( \mathrm{Re}\{S_k\} \cos\left( \frac{2\pi k t}{T_s} \right) - \mathrm{Im}\{S_k\} \sin\left( \frac{2\pi k t}{T_s} \right) \right)$$

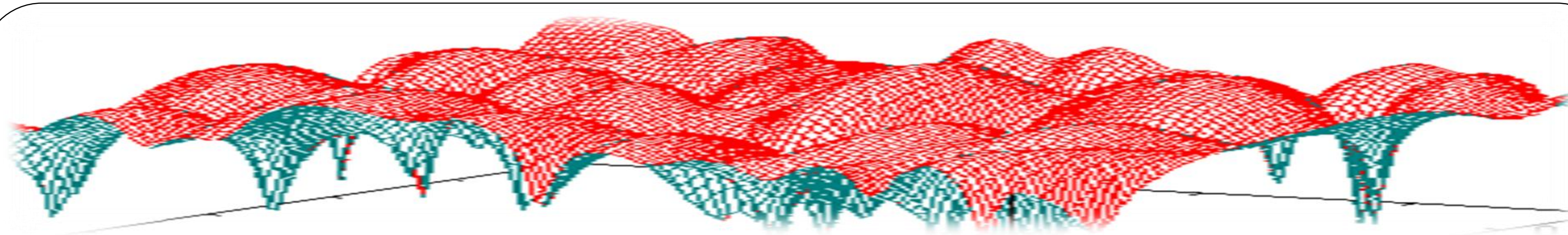
# Single-User OFDM

**In this section,**
**we shall focus on the**
**Single-user case of OFDM.**

# Motivation

## Why do we need OFDM?

- First, we study the wireless channel.
- There are a couple of difficult problems in communication system over wireless channel.
- Also want to achieve high data rate (throughput)



# ECS455: Chapter 5

## OFDM

5.1 Wireless Channel (A Revisit)

Dr.Prapun Suksompong

www.prapun.com

**Office Hours:**

| | | |
|---|---|---|
| Library (Rangsit) | Mon | 16:20-16:50 |
| BKD 3601-7 | Wed | 9:20-11:20 |

# Single Carrier Digital Transmission

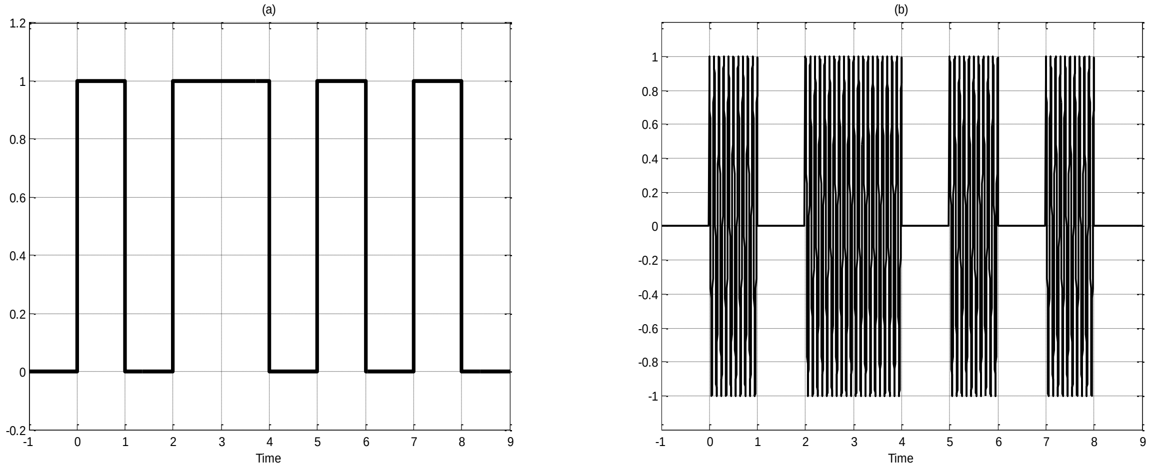
- Baseband:

$$s(t)=\sum_{k=0}^{N-1} s_k\, p(t-kT_s)$$

$$p(t)=1_{[0,T_s)}(t)=\begin{cases}1, & t\in[0,T_s)\\ 0, & \text{otherwise.}\end{cases}$$

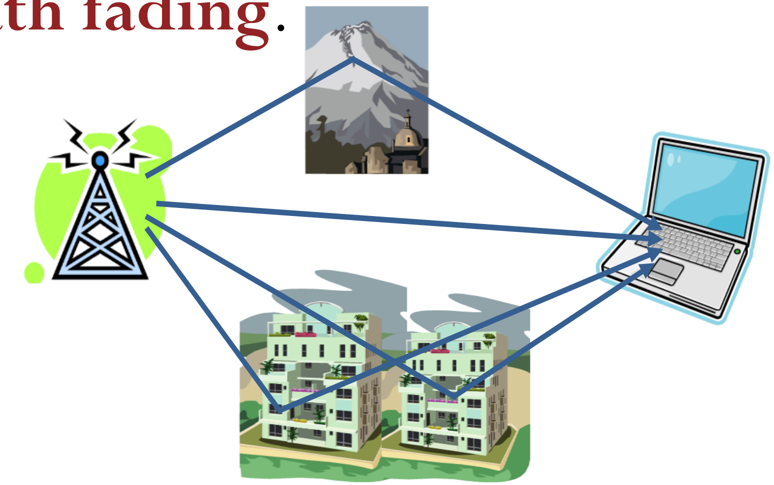
- Passband:

$$x(t)=\mathrm{Re}\left\{s(t)e^{j2\pi f_c t}\right\}$$

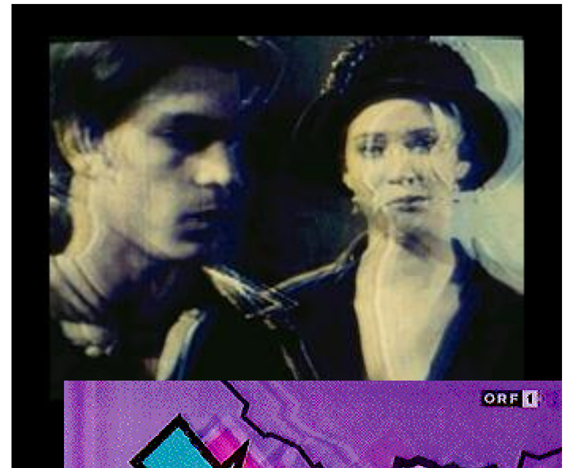


# Multipath Propagation

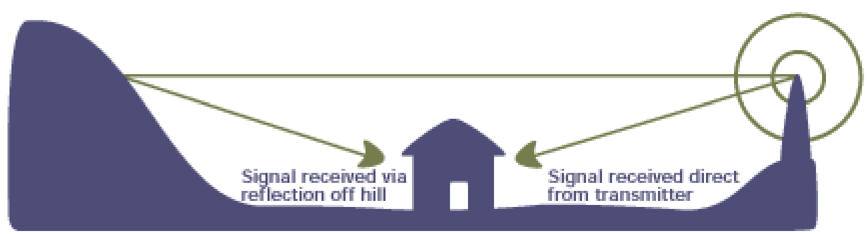
- In a wireless mobile communication system, a transmitted signal propagating through the wireless channel often encounters multiple reflective paths until it reaches the receiver
- We refer to this phenomenon as **multipath propagation** and it causes fluctuation of the amplitude and phase of the received signal.
- We call this fluctuation **multipath fading**.



# Similar Problem: Ghosting

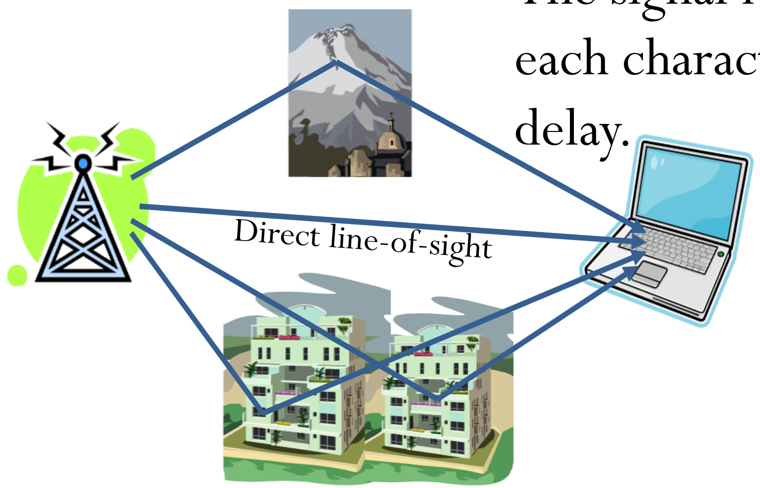






# Wireless Comm. and Multipath Fading

The signal received consists of a number of reflected rays, each characterized by a different amount of attenuation and delay.

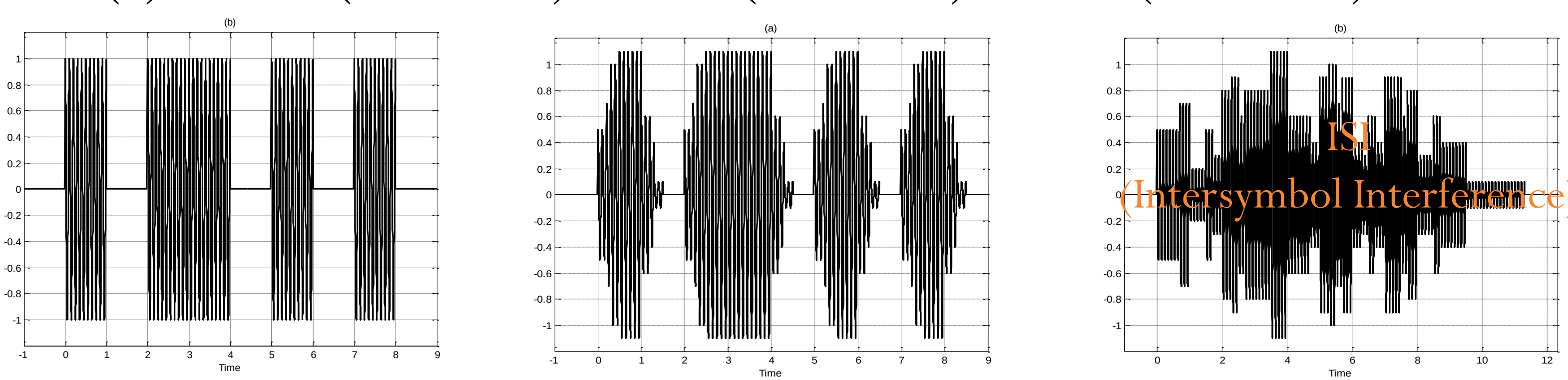



$$r(t) = x(t) * h(t) + n(t) = \sum_{i=0}^{v} \beta_i x(t - \tau_i) + n(t)$$

$$h(t) = \sum_{i=0}^{v} \beta_i \delta(t - \tau_i)$$

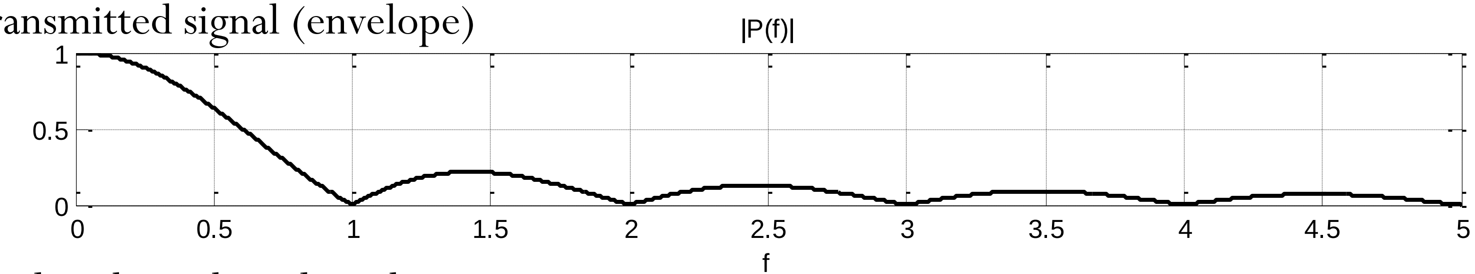
$$h_1(t) = 0.5\delta(t) + 0.2\delta(t - 0.2T_s) + 0.3\delta(t - 0.3T_s) + 0.1\delta(t - 0.5T_s)$$

$$h_2(t) = 0.5\delta(t) + 0.2\delta(t - 0.7T_s) + 0.3\delta(t - 1.5T_s) + 0.1\delta(t - 2.3T_s)$$

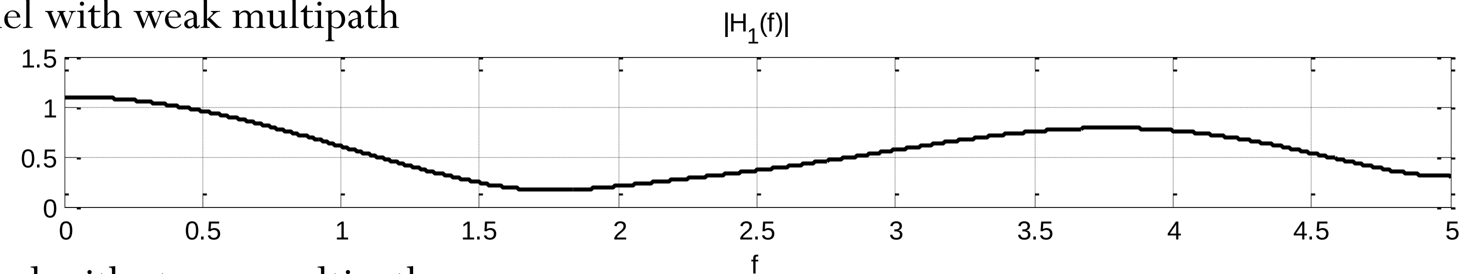


# Frequency Domain

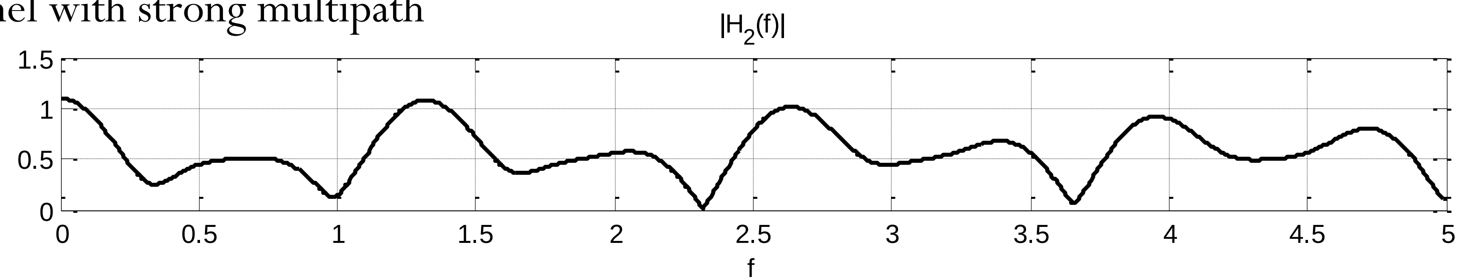
The transmitted signal (envelope)




Channel with weak multipath




Channel with strong multipath



# Observation

- Delay spread causes ISI
- Observation: A general rule of thumb is that a delay spread of less than 5 or 10 times the symbol width will not be a significant factor for ISI.
- Solution: The ISI can be mitigated by reducing the symbol rate and/or including sufficient guard times between symbols.

# COST 207 Channel Model

- Based on channel measurements with a bandwidth of 8–10MHz in the 900MHz band used for 2G systems such as GSM.

| Path # | Rural Area (RA) | | Typical Urban (TU) | | Bad Urban (BU) | | Hilly Terrain (HT) | |
|---|---|---|---|---|---|---|---|---|
| | Delay | Power | Delay | Power | Delay | Power | Delay | Power |
| | (µs) | (dB) | (µs) | (dB) | (µs) | (dB) | (µs) | (dB) |
| 1 | 0 | 0 | 0 | −3 | 0 | −2.5 | 0 | 0 |
| 2 | 0.1 | −4 | 0.2 | 0 | 0.3 | 0 | 0.1 | −1.5 |
| 3 | 0.2 | −8 | 0.5 | −2 | 1.0 | −3 | 0.3 | −4.5 |
| 4 | 0.3 | −12 | 1.6 | −6 | 1.6 | −5 | 0.5 | −7.5 |
| 5 | 0.4 | −16 | 2.3 | −8 | 5.0 | −2 | 15.0 | −8.0 |
| 6 | 0.5 | −20 | 5.0 | −10 | 6.6 | −4 | 17.2 | −17.7 |

[Fazel and Kaiser, 2008, Table 1-1]

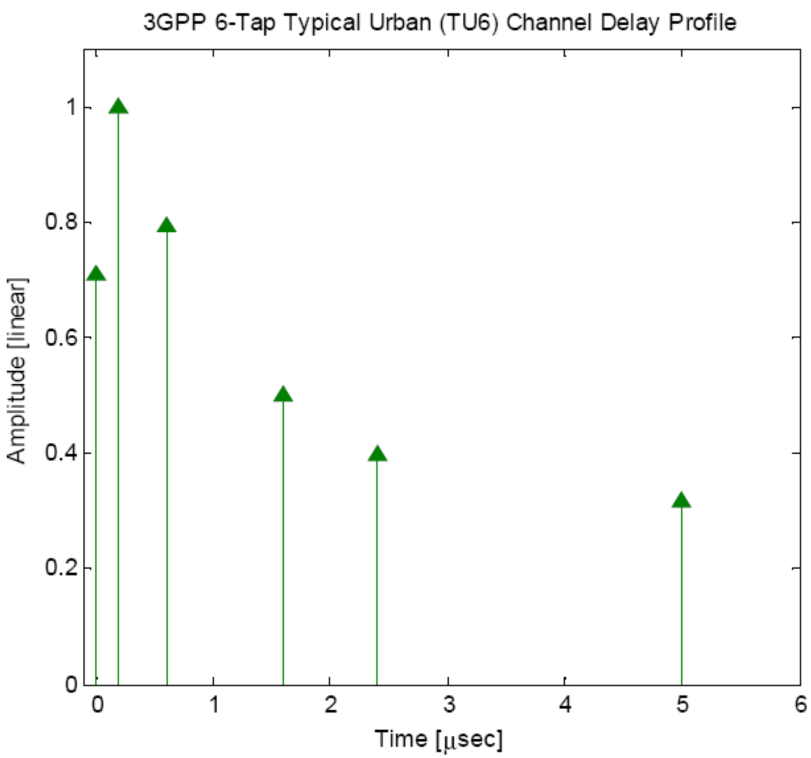
# 3GPP LTE Channel Modelss

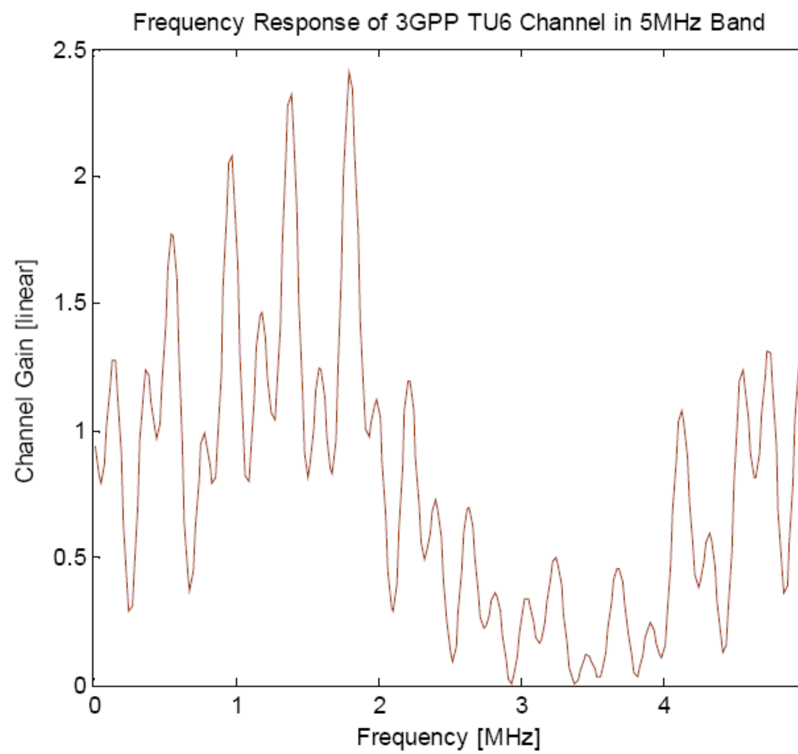
| Path number | Extended Pedestrian A (EPA) | | Extended Vehicular A (EVA) | | Extended Typical Urban (ETU) | |
|---|---|---|---|---|---|---|
| | Delay (ns) | Power (dB) | Delay (ns) | Power (dB) | Delay (ns) | Power (dB) |
| 1 | 0 | 0 | 0 | 0 | 0 | −1 |
| 2 | 30 | −1 | 30 | −1.5 | 50 | −1 |
| 3 | 70 | −2 | 150 | −1.4 | 120 | −1 |
| 4 | 90 | −3 | 310 | −3.6 | 200 | 0 |
| 5 | 110 | −8 | 370 | −0.6 | 230 | 0 |
| 6 | 190 | −17.2 | 710 | −9.1 | 500 | 0 |
| 7 | 410 | −20.8 | 1090 | −7 | 1600 | −3 |
| 8 | | | 1730 | −12 | 2300 | −5 |
| 9 | | | 2510 | −16.9 | 5000 | −7 |

[Fazel and Kaiser, 2008, Table 1-3]

# 3GPP 6-tap typical urban (TU6)

- Delay profile and frequency response of 3GPP 6-tap typical urban (TU6) Rayleigh fading channel in 5 MHz band.




[3GPP TS 45.005 – 3GPP; Technical Specification Group GSM/EDGE Radio Access Network; Radio Transmission and Reception (Release 7)]

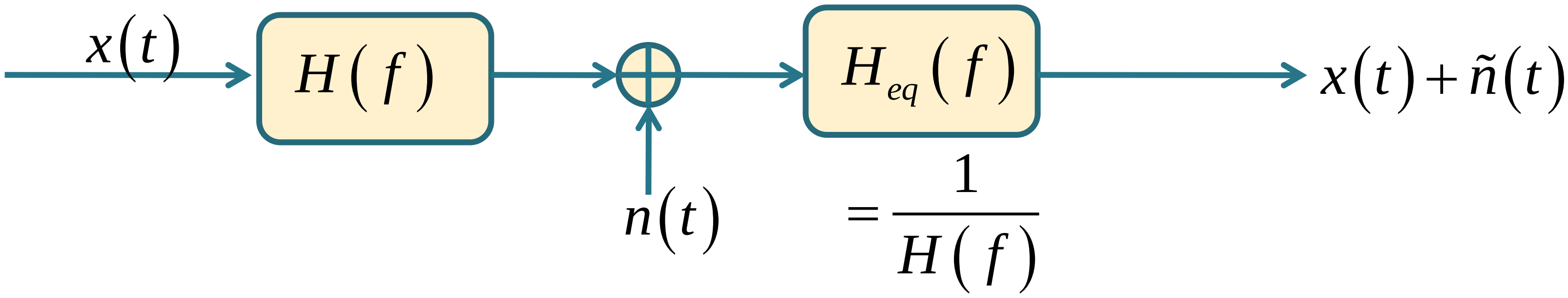
# Equalization

- Chapter 11 of [Goldsmith, 2005]
- In a broad sense, **equalization** defines any signal processing technique used at the *receiver* to alleviate the ISI problem caused by delay spread. [Goldsmith, 2005]
- Higher data rate applications are more sensitive to delay spread, and generally require high-performance equalizers or other ISI mitigation techniques.
- Signal processing can also be used at the *transmitter* to make the signal less susceptible to delay spread.
  - Ex. spread spectrum and multicarrier modulation

# Equalizer design

- Need to **balance ISI mitigation with noise enhancement**
  - Both the signal and the noise pass through the equalizer
- Nonlinear equalizers suffer less from noise enhancement than linear equalizers, but typically entail higher complexity.
- Most equalizers are implemented digitally after A/D conversion
  - Such filters are small, cheap, easily tuneable, and very power efficient.
- The *optimal* equalization technique is **maximum likelihood sequence estimation (MLSE)**.
  - Unfortunately, the complexity of this technique (even when using **Viterbi algorithm**) grows exponentially with the length of the delay spread, and is therefore ***impractical*** on most channels of interest.

# Simple Analog Equalizer



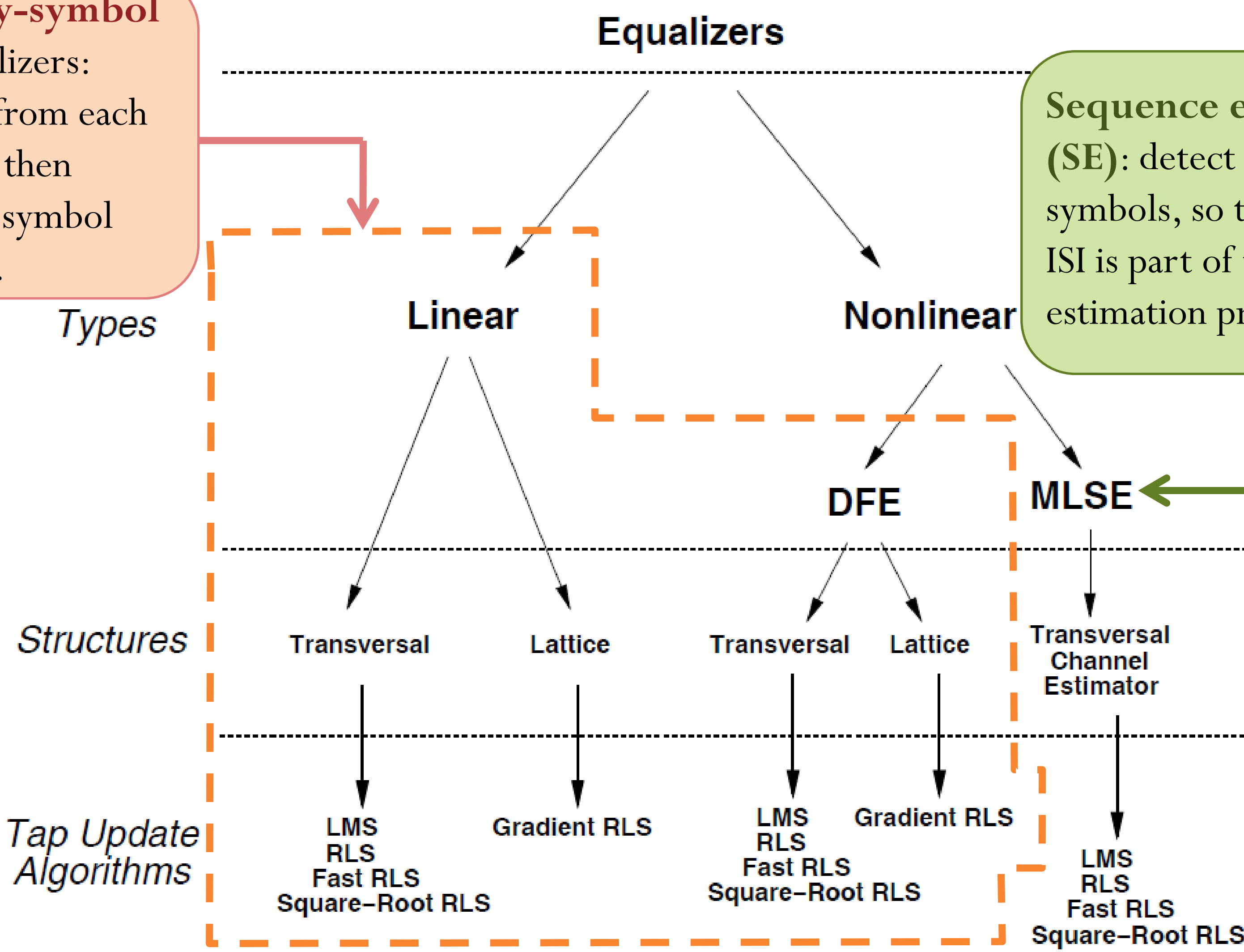

- Attempt to *remove all ISI*
- Disadvantages:
  - If some frequencies in the channel frequency response $H(f)$ are greatly attenuated, the equalizer $H_{eq}(f) = 1/H(f)$ will greatly enhance the noise power at those frequencies.
  - If the channel frequency response $H(f)$ has a spectral null ($= 0$ for some frequency), then the power of the new noise is infinite.
- Even though the ISI effects are (completely) removed, the equalized system will perform poorly due to its greatly reduced SNR.

# Linear vs. Non-linear Equalizers

- Need to balance mitigation of the effects of ISI with maximizing the SNR of the post-equalization signal.
- **Linear** digital equalizers
  - In general work by inverting the channel frequency response
  - Easy to implement and to understand conceptually
  - Typically suffer from more noise enhancement
  - Not used in most wireless applications
- **Nonlinear** equalizers
  - Do not invert the channel frequency response
  - Suffer much less from noise enhancement
  - **Decision-feedback equalization (DFE)** is the most common
    - Fairly simple to implement and generally performs well.

# Equalizer Types

**Symbol-by-symbol (SBS)** equalizers: remove ISI from each symbol and then detect each symbol individually.




**Sequence estimators (SE)**: detect sequences of symbols, so the effect of ISI is part of the estimation process.

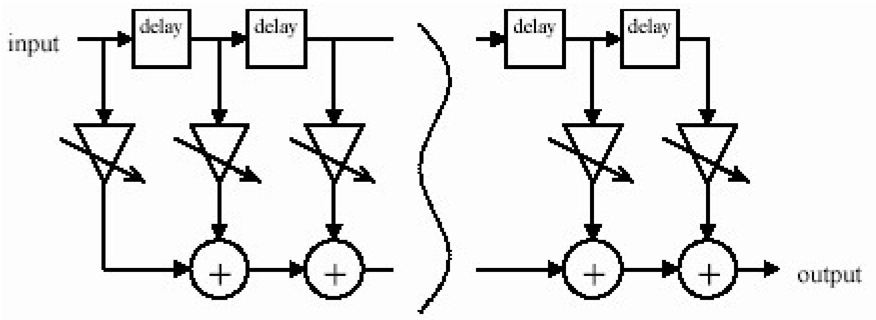
[Goldsmith, 2005, Fig. 11.2]

# Transversal Structure

- Linear and nonlinear equalizers are typically implemented using a transversal or lattice structure.
- The transversal structure is a filter with $N-1$ delay elements and $N$ taps with tunable complex weights.

$$H_{eq}(z) = \sum_{i=-L}^{L} w_i z^{-i}$$

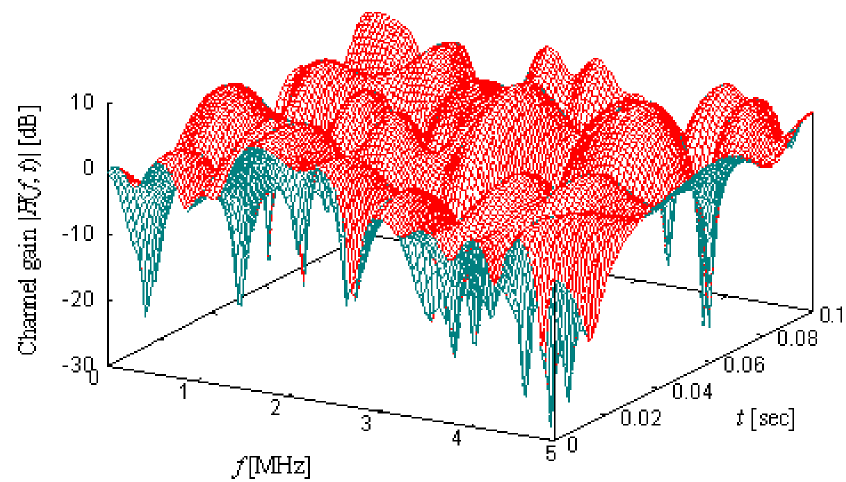
$$N = 2L + 1$$




- The length of the equalizer $N$ is typically dictated by implementation considerations
  - Large $N$ usually entails higher complexity.

# Time-varying Multipath Channel

- Impulse Response: $h(\tau,t) = \sum_{i=0}^{L-1} \beta_i(t)\delta(\tau - \tau_i)$
  - $L$ = number of resolvable paths
  - $\beta_i(t)$ = complex-valued path gain of the $i$th path
    - Usually assumed to be independent complex Gaussian processes resulting in Rayleigh fading because each resolvable path is the contribution of a different group of many irresolvable paths.
  - $\tau_i$ = time delay of the $i$th path
- Transfer function: $H(f,t)$




L = 16-path exponential power delay profile with a decay factor of 1.0 dB and a time delay separation of 150 ns between adjacent paths (corresponding to the rms delay spread of 0.52 μs). 5 GHz carrier frequency and 4 km/h terminal speed.

[Adachi, Garg, Takaoka, and Takeda, 2005, Figure 2]

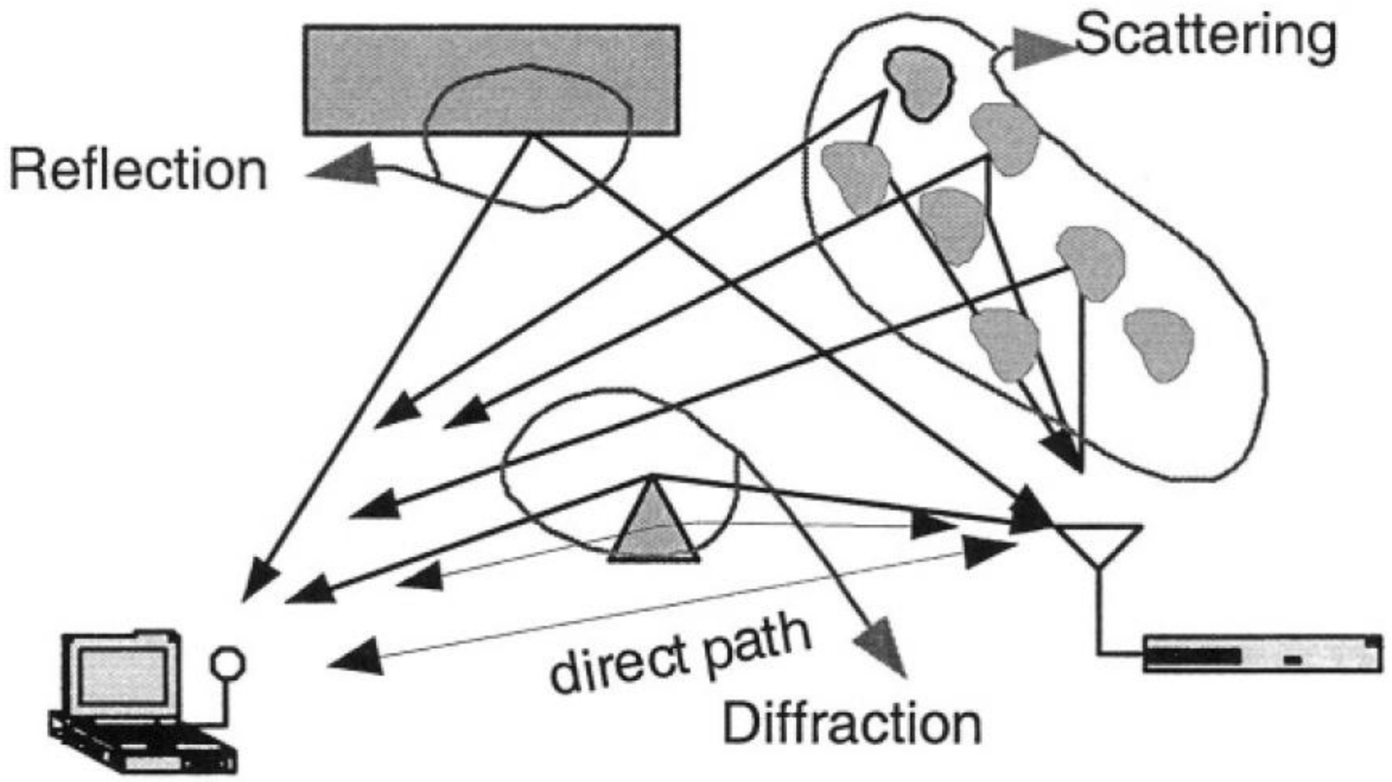
# Adaptive Equalization

- Equalizers must typically have an ***estimate*** of the channel (impulse or frequency response)
  - Since the wireless channel varies over time, the equalizer must
    - learn the frequency or impulse response of the channel (**training**)
    - and then update its estimate of the frequency response as the channel changes
- The process of equalizer training and tracking is often referred to as **adaptive equalization**.
- **Blind equalizers** do not use training
  - Learn the channel response via the detected data only

# Equalization for Digital Cellular Telephony

- GSM
  - Use adaptive equalizer
  - Equalize echos up to 16 ms after the first signal received
    - Correspond to 4.8 km in distance.
    - One bit period is 3.69 ms. Hence, echos with about 4 bit lengths delay can be compensated
- The direct sequence spreading employed by CDMA (IS-95) obviates the need for a traditional equalizer.
- If the transmission bandwidth is large (for example 20 MHz), the complexity of straightforward high-performance equalization starts to become a serious issue.

# Wireless Propagation




[Bahai, 2002, Fig. 2.1]

# Three steps towards modern OFDM

1. To mitigate multipath problem
   → Use multicarrier modulation (FDM)
2. To gain spectral efficiency
   → Use orthogonality of the carriers
3. To achieve efficient implementation
   → Use FFT and IFFT